Section 2.4 – Complex Numbers

Example: Solve $x^2 + 1 = 0$.

$x^2 = -1$

$x = \pm\sqrt{-1}$?? what do we do?

To overcome the inability to solve this in the real number system, a COMPLEX NUMBER SYSTEM was created.

Complex Numbers

A *complex number* has both a REAL component and an IMAGINARY component.

A complex number is written in standard form as $a+bi$, where a is a *real part* and bi is the *imaginary part* (b alone is a real number).

Principal Square Roots of Negative Numbers

If a is a positive number, the principal square root of the negative number –a is defined as:

$$\sqrt{-a} = i\sqrt{a}.$$

Examples: Write the complex number in standard form.

1. $\sqrt{-3}\sqrt{-12} = (i\sqrt{3})(i\sqrt{12}) = i^2\sqrt{36} = 6i^2 = -6$

2. $\sqrt{-48} - \sqrt{-27} = i\sqrt{48} - i\sqrt{27}$

$= 4i\sqrt{3} - 3i\sqrt{3} = i\sqrt{3}$

Section 2.4 – Complex Numbers

Operations with Complex Numbers

Addition and Subtraction:

Examples:

1. $(4+7i)+(1-6i)=$ $5+i$

2. $(1+2i)-(4+2i)=$ -3

3. $3i-(-2+3i)-(2+5i)=$

$= 3i+2-3i-2-5i$

$= -5i$

4. $(3+2i)+(4-i)-(7+i)=$

$= 3+2i+\cancel{4}-i-\cancel{7}-i$

$= 0$

RECALL: $i^1=i$ **and** $i^2=-1$**, so** $i^3=i\cdot i^2=i\cdot(-1)=-i$ **and** $i^4=i^2\cdot i^2=(-1)(-1)=1$

Examples (Multiplying):

1. $4(-2+3i)=$ $-8+12i$

2. $(2-i)(4-3i)=$ $8-6i-4i+3i^2$

$= 8-10i+3(-1)$

$= 5-10i$

3. $(3+2i)(3-2i)=$ $9-6i+6i-4i^2$

$= 9-4(-1) = 13$

Section 2.4 – Complex Numbers

Solve the following equations.

1. $x^2 + 4 = 0$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

$$= \pm i\sqrt{4}$$

$$= \pm 2i$$

2. $3x^2 - 2x + 5 = 0$

Doesn't factor)

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2-4(3)(5)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4-60}}{6} = \frac{2 \pm \sqrt{-56}}{6}$$

$$= \frac{2 \pm 2i\sqrt{14}}{6} = \frac{1 \pm i\sqrt{14}}{3}$$

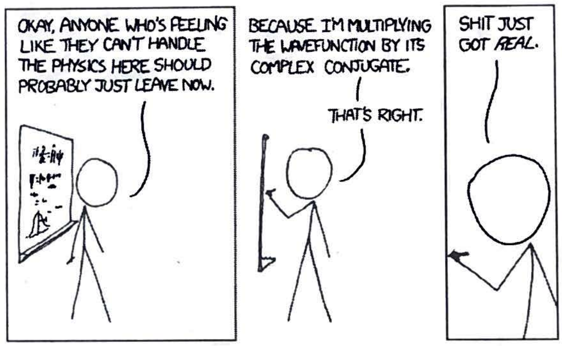
$$= \frac{1}{3} \pm \frac{i\sqrt{14}}{3}$$

Complex Conjugates

The conjugate of a complex number of the form $a+bi$ is $a-bi$.

Example: Multiply $4-3i$ by its complex conjugate.

$$(4-3i)(4+3i)$$
$$=16-12i+12i-9i^2$$
$$=16+9=25$$



Example: Write the quotient of the following complex number in standard form ($a + bi$).

$$\frac{2+3i}{4-2i}=\left(\frac{4+2i}{4+2i}\right)=\frac{8+4i+12i+6i^2}{16+8i-8i-4i^2}$$
$$=\frac{8+16i-6}{16+4}=\frac{2+16i}{20}=\frac{1}{10}+\frac{4i}{5}$$